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Partitioning of vorticity in bimineralic rocks

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Abstract

A model for the rotational behaviour of flow in bimineralic rocks based on the concept of a multiphase continuum is developed here. An additive relation with respect to the vorticity is assumed. The ideal linear viscous bimineralic rocks are defined such that (i) both the bulk rock and the constituent phases have linear viscous rheology, (ii) the phases are uniformly mixed and distributed in the rock, and (iii) no segregation occurs during the deformation. The model reveals the partitioning of vorticity between the two phases in the ideal linear viscous bimineralic rocks. The model shows that there are two modes of flow behaviour. In mode 1 there is neither partitioning of strain rate nor vorticity; in mode 2 partitioning occurs for strain rate and vorticity. Mode 1 behaviour corresponds to upper bound behaviour and mode 2 to the lower bound behaviour. There are two solutions for the partitioning of vorticity in mode 2: type 1 and type 2 partitioning. In the type 1 partitioning the more viscous phase is more rotational than the less viscous phase, while in the type 2 partitioning the former is less rotational than the latter. In a special case where flow occurs by simple shearing and where the more viscous phase behaves as rigid particles, the following two cases occur: (1) the more viscous phase rotates at twice the rate of the bulk rock (type 1 partitioning), (2) the more viscous phase does not rotate at all with respect to the external reference coordinate (type 2 partitioning). The present model alone cannot predict which type will occur and further theoretical and experimental work is needed.

1. Introduction

The present paper contains an extension of Takeda (1998), in which a model for flow in bimineralic rocks was developed. The model is based on the concept of a multiphase continuum. If a polymineralic rock is considered as a multiphase continuum, additive relations for several physical quantities may be assumed, that is, any quantity for the bulk rock is equal to the sum of those for the constituent phases. The additive relations are held as internal restrictions on the flow behaviour of polymineralic rocks. Takeda (1998) investigated the partitioning of deformation rate into the constituent phases in bimineralic rocks by means of several additive relations under a number of assumptions. Three main additive relations were considered: deformation rate, which results from an additive relation of linear momentum; viscous stress; and entropy production rate (or dissipative energy). The main assumptions made are that (i) both the bulk rock and the constituent phases have linear viscous rheology, (ii) the phases are uniformly mixed and distributed evenly in the rock, and (iii) no temporal change occurs in the volume proportion of each phase during the deformation (i.e. no segregation). Here we call such a modelled bimineralic rock the ideal linear viscous bimineralic rock. In Takeda (1998) two possible types of partitioning behaviour of deformation rate have been determined for an ideal linear viscous bimineralic rock: mode 1 behaviour and mode 2 behaviour. In the present paper it is shown that mode 1 behaviour and mode 2 behaviour correspond to the Voigt bound behaviour and the Reuss bound behaviour, respectively (e.g. Hill, 1965).

Since a velocity gradient tensor can be decomposed into a deformation rate tensor and a vorticity vector (or spin tensor), a complete description of a flow field requires knowledge of both the deformation rate and the vorticity. Hence, it is attempted to determine the partitioning of vorticity between the two constituent phases in an ideal linear viscous bimineralic rock.

2. Flow of an ideal linear viscous bimineralic rock

Takeda (1998) derived several intrinsic relations for flow of ideal linear viscous bimineralic rocks. The derived relations contain two types of equations; one is for the bulk rock viscosity, the other for the partitioning of deformation rate between the two phases.

The bulk rock viscosity is given by either (Takeda, 1998)

$$\mu^* = (1 - b)\phi_1 + b \tag{1}$$

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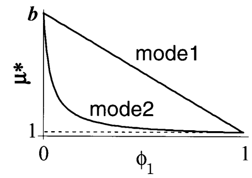

Fig. 1. Schematic curves showing the relation between the normalized bulk rock viscosity ($\mu^* = \mu/\mu_1$) and the volume fraction of less viscous phase (ϕ_1) predicted by the model. Mode 1 behavior shows a linear relationship, mode 2 behavior shows a non-linear relationship. b is a viscosity contrast between the phases (μ/μ_1). μ is a bulk rock viscosity. μ_1 is a viscosity of less viscous phase. μ_2 is a viscosity of more viscous phase.

or

$$\mu^* = \frac{[a - a\phi_1 + \phi_I]^2 b}{a^2 + (b - a^2)\phi_1}, \tag{2}$$

where a is the density contrast defined by $a = \gamma_2/\gamma_1$; b is the viscosity contrast defined by $b = \mu_2/\mu_1$; μ^* is the normalized bulk rock viscosity defined by $\mu^* = \mu_b/\mu_1$; γ_1 and γ_2 are densities of the more viscous phase and the less viscous phase, respectively; μ_1 and μ_2 are viscosities of the more viscous phase and of the less viscous phase, respectively; μ_b is the viscosity of the bulk rock, and ϕ_1 is the volume fraction of the less viscous phase. The presence of two equations for the bulk rock viscosity implies that there are two types of flow behaviour for an ideal linear viscous bimineralic rock. Eq. (1) describes mode 1 behaviour and Eq. (2) mode 2 behaviour of Takeda (1998) (Fig. 1).

The deformation rate of each phase for mode 1 behaviour is equal and given by

$$D_{ij}^1 = D_{ij}^2 = D_{ij}^b. \tag{3}$$

Those for mode 2 behaviour are

$$D_{ij}^1 = P_1 D_{ij}^b \quad \text{and} \quad D_{ij}^2 = P_2 D_{ij}^b, \tag{4}$$

where

$$P_1 = \frac{b(a - a\phi_1 + \phi_I)}{a^2 + (b - a^2)\phi_I}, \tag{5}$$

$$P_2 = \frac{a(a - a\phi_I + \phi_I)}{a^2 + (b - a^2)\phi_I}, \tag{6}$$

D_{ij}^b is a deformation rate tensor of the bulk rock, D_{ij}^1 is a deformation rate tensor of the less viscous phase, and D_{ij}^2 is a deformation rate tensor of the more viscous phase. P_1 and P_2 are partitioning coefficients of deformation rate for each phase.

Eq. (3) indicates that mode 1 behaviour corresponds to the case of uniform deformation rate (strain rate). When the density contrast between the two phases is unity, Eq. (4) implies that viscous stresses of the two phases are related by

$$T_{ij}^1 = \mu_1 D_{ij}^1 = \mu_2 D_{ij}^2 = T_{ij}^2, \tag{7}$$

where T_{ij}^1 and T_{ij}^2 are viscous stress tensors of the less viscous phase and more viscous phase, respectively, while μ_1 and μ_2 are viscosities of the less viscous phase and the more viscous phase, respectively. Therefore, mode 2 behaviour corresponds to the case of a uniform viscous stress when densities of the two phases are the same. Consequently, mode 1 behaviour corresponds with the so-called Voigt bound and mode 2 behaviour corresponds with the so-called Reuss bound when no density contrast is present (e.g. Hill, 1965). This is also evident from the fact that when two phases have the same density Eq. (2) can be rewritten as

$$\mu_b = \left(\frac{\phi_1}{\mu_1} + \frac{\phi_2}{\mu_2} \right)^{-1},$$

where ϕ_1 and ϕ_2 are volume fractions of the less viscous phase and the more viscous phase, respectively, and where $\phi_1 + \phi_2 = 1$.

3. Partitioning of vorticity

The previous section explained partitioning of deformation rate between the two phases for ideal linear viscous bimineralic rocks. In order to better understand the behaviour of the flow in two-phase rocks the partitioning of vorticity is considered for ideal linear viscous bimineralic rocks. However, it is difficult to obtain a general solution because there are too many parameters that cannot be constrained. For example, six parameters are needed to describe the vorticity vectors of the two phases. It is assumed, therefore, that vorticity vectors of the two phases are parallel to each other in space, which is referred to as 'homo-axial flow' in this paper. This assumption is made rather intuitively but is considered to be a reasonable one as a first step of the present theoretical approach.

As mentioned above, for ideal linear viscous bimineralic rocks it is assumed that the two phases are uniformly mixed and that the volume fraction is kept constant. The total linear momentum of the bulk rock in a given region is the sum of the linear momentum of two phases as

$$\gamma_b \mathbf{V}_b = \phi_1 \gamma_1 \mathbf{V}_1 + \phi_2 \gamma_2 \mathbf{V}_2, \tag{8}$$

where $\mathbf{V}_b$ and γ_b are the velocity vector and the density of the bulk rock, $\mathbf{V}_1$ and $\mathbf{V}_2$ are velocity vectors of the less viscous phase and of the more viscous phase, respectively, and γ_1 and γ_2 are densities of the less viscous phase and of the more viscous phase. If we further assume that two phases have the same density, Eq. (8) is reduced to

$$\mathbf{V}_b = \phi_1 \mathbf{V}_1 + \phi_2 \mathbf{V}_2. \tag{9}$$

Taking the curl of Eq. (9) produces

$$\boldsymbol{\omega}_b = \phi_1 \boldsymbol{\omega}_1 + \phi_2 \boldsymbol{\omega}_2, \tag{10}$$

where $\boldsymbol{\omega}_b$ is the vorticity vector of the bulk rock and $\boldsymbol{\omega}_1$ and $\boldsymbol{\omega}_2$ are vorticity vectors of the less viscous phase and the more viscous phase, respectively.

In homo-axial flow the vorticity vector for the less viscous phase $\boldsymbol{\omega}_1$ and the more viscous phase $\boldsymbol{\omega}_2$ are given by

$$\boldsymbol{\omega}_1 = \alpha\boldsymbol{\omega}_b, \tag{11}$$

$$\boldsymbol{\omega}_2 = \beta\boldsymbol{\omega}_b, \tag{12}$$

where α and β are vorticity partitioning coefficients for the less viscous phase and the more viscous phase, respectively. Substituting Eqs. (11) and (12) into Eq. (10), we have

$$\phi_1\alpha + \phi_2\beta = 1. \tag{13}$$

In order to determine the partitioning of vorticity it is necessary to derive another additive relation. The additive relation for acceleration vectors is obtained by taking the time derivate of Eq. (9) as

$$\dot{\mathbf{V}}_b = \phi_1\dot{\mathbf{V}}_1 + \phi_2\dot{\mathbf{V}}_2, \tag{14}$$

where $\dot{\mathbf{V}}_b$ is an acceleration vector for the bulk rock, $\dot{\mathbf{V}}_1$ and $\dot{\mathbf{V}}_2$ are acceleration vectors for the less viscous phase and for the more viscous phase, respectively. Taking the divergence of Eq. (14), we have

$$\left[D_{ij}^{b}D_{ij}^{b} - \frac{1}{2}\left|\boldsymbol{\omega}_b\right|^2\right] = \phi_1\left[D_{ij}^{1}D_{ij}^{1} - \frac{1}{2}\left|\boldsymbol{\omega}_1\right|^2\right] + \phi_2\left[D_{ij}^{2}D_{ij}^{2} - \frac{1}{2}\left|\boldsymbol{\omega}_2\right|^2\right], \tag{15}$$

where the following relation of divergence of an acceleration vector for incompressible flow is used (e.g. Truesdell and Toupin, 1960)

$$\nabla\cdot\dot{\mathbf{V}} = D_{ij}D_{ij} - \frac{1}{2}|\boldsymbol{\omega}|^2, \tag{16}$$

where $\dot{\mathbf{V}}$ is the acceleration vector, D_{ij} is the deformation rate tensor, $|\boldsymbol{\omega}|$ is the magnitude of the vorticity vector. It should be noticed that each phase in the ideal linear viscous bimineralic rock is assumed to be incompressible and also that the bulk rock is incompressible because the volume fraction of the two phases is assumed to be constant. Accordingly, substituting Eqs. (11) and (12) into Eq. (15), we have

$$2(D_{ij}^{b}D_{ij}^{b} - \phi_1 D_{ij}^{1}D_{ij}^{1} - \phi_2 D_{ij}^{2}D_{ij}^{2}) = (1 - \phi_1\alpha^2 - \phi_2\beta^2)|\boldsymbol{\omega}_b|^2. \tag{17}$$

Consequently, vorticity partitioning coefficients for each phase, α and β, can be determined from Eqs. (13) and (17). It will be shown below that the partitioning of vorticity is derived for mode 1 and mode 2 as in the case of the partitioning of deformation rates.

3.1. The partitioning of vorticity in mode 1 behaviour

Deformation rate of each phase in mode 1 behaviour is given by Eqs. (3). Substituting Eqs. (3) into Eq. (17), we have

$$\phi_1\alpha^2 + \phi_2\beta^2 = 1. \tag{18}$$

From Eqs. (13) and (18) vorticity partitioning coefficients of each phase are equal to unity since

$$\alpha = \beta = 1. \tag{19}$$

This means that there is no partitioning of vorticity in mode 1 behaviour. Hence, it can be said that where there is no partitioning of deformation rate, there is no partitioning of vorticity either.

3.2. The partitioning of vorticity in mode 2 behaviour

Deformation rates of each phase in mode 2 behaviour are given by Eqs. (4)–(6). Substituting these into Eq. (17), we have

$$2(D_{ij}^{b}D_{ij}^{b} - \phi_1 P_1^2 D_{ij}^{b}D_{ij}^{b} - \phi_2 P_2^2 D_{ij}^{b}D_{ij}^{b}) = (1 - \phi_1\alpha^2 - \phi_2\beta^2)|\boldsymbol{\omega}_b|^2. \tag{20}$$

Using a kinematic vorticity number for the bulk rock, ϖ_{kb}, which is defined as (e.g. Truesdell, 1953)

$$\varpi_{kb} = \frac{|\boldsymbol{\omega}_b|}{\sqrt{2D_{ij}^{b}D_{ij}^{b}}},$$

Eq. (20) can be rewritten as

$$\frac{1}{(\varpi_{kb})^2}(1 - \phi_1 P_1^2 - \phi_2 P_2^2) = (1 - \phi_1\alpha^2 - \phi_2\beta^2). \tag{21}$$

Accordingly, from Eqs. (13) and (21) vorticity partitioning coefficients of each phase in mode 2 are obtained as

$$[\alpha_I,\ \beta_I] = \left[1 + \frac{1}{\varpi_{kb}}\cdot\frac{(1-\phi_1)(1-b)}{1+(b-1)\phi_1},\ 1 - \frac{1}{\varpi_{kb}}\cdot\frac{\phi_1(1-b)}{1+(b-1)\phi_1}\right] \tag{22}$$

and

$$[\alpha_{II},\ \beta_{II}] = \left[1 - \frac{1}{\varpi_{kb}}\cdot\frac{(1-\phi_1)(1-b)}{1+(b-1)\phi_1},\ 1 + \frac{1}{\varpi_{kb}}\cdot\frac{\phi_1(1-b)}{1+(b-1)\phi_1}\right]. \tag{23}$$

It is noted that there are two solutions as above, i.e. there can be two ways for partitioning of vorticity in mode 2 behaviour. The type of partitioning of vorticity given by matrix (22) is referred to as type 1 partitioning and that given by matrix (23) as type 2 partitioning hereafter. It is evident that partitioning of vorticity is dependent on the kinematic vorticity number of the bulk rock in both types. This means that the partitioning of vorticity is dependent not only on material properties such as the viscosity contrast and the volume fraction but also on the kinematics of bulk rock flow, which is in contrast to the case of partitioning of deformation rate.

In type 1 partitioning the vorticity partitioning coefficient

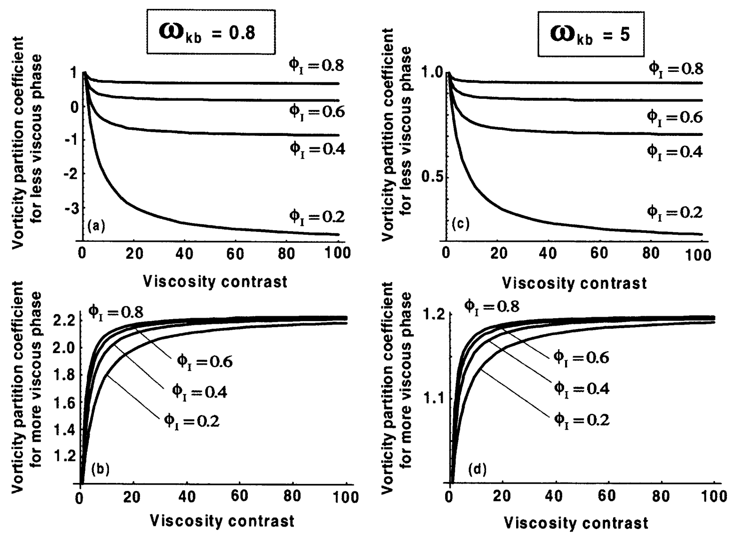

Fig. 2. The relationships between the vorticity partition coefficients of each constituent phase and the viscosity contrast for different volume fractions at various kinematical vorticity number for the bulk rock (ω_{kb}) in type 1 partititioning: (a) for the less viscous phase at $\omega_{kb} = 0.8$, (b) for the more viscous phase at $\omega_{kb} = 0.8$, (c) for the less viscous phase at $\omega_{kb} = 5$, (d) for the more viscous phase at $\omega_{kb} = 5$.

of the less viscous phase, α_1, is always less than unity because the second term on the left side in matrix (22) is negative. Conversely, the vorticity partitioning coefficient of the more viscous phase, β_1, in type 2 partitioning is always greater than unity because the second term on the right side in matrix (22) is negative. Hence, the more viscous phase is more rotational than the less viscous phase in type 1 partitioning. In addition, with decreasing kinematic vorticity number of the bulk rock and increasing viscosity contrast between the two phases, the vorticity partitioning coefficient of the more viscous phase increases while that of the less viscous phase decreases (Fig. 2). In particular it should be noted that the vorticity partitioning coefficient of the less viscous phase can be negative. Furthermore, the vorticity partitioning coefficients for both phases, α_I and β_I, increase with increasing volume fraction of the less viscous phase.

On the other hand, partitioning of vorticity in type 2 partitioning is almost opposite in behaviour to that of type 1 partitioning. Thus, the less viscous phase is more rotational than the more viscous phase in type 2. With decreasing kinematic vorticity number of the bulk rock and increasing viscosity contrast, the vorticity partitioning coefficient of the less viscous phase increases while the vorticity partitioning coefficient of the more viscous phase decreases (Fig. 3); the vorticity partitioning coefficient of the more viscous phase can be negative. Furthermore, both vorticity partitioning coefficients, α_{II} and β_{II}, decrease with increasing volume fraction of the less viscous phase (Fig. 3).

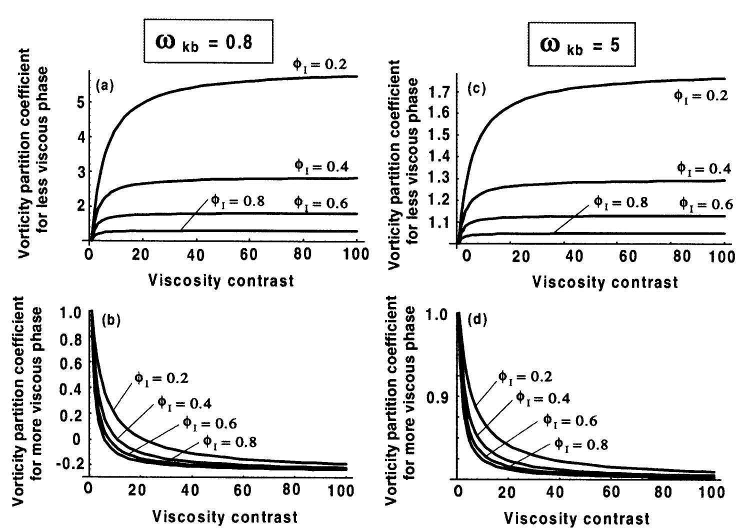

Fig. 3. The relationships between the vorticity partition coefficient of each constituent phase and the viscosity contrast for different volume fractions at various kinematical vorticity number for the bulk rock (ω_{kb}) in type 2 partititioning: (a) for the less viscous phase at $\omega_{kb} = 0.8$, (b) for the more viscous phase at $\omega_{kb} = 0.8$, (c) for the less viscous phase at $\omega_{kb} = 5$, (d) for the more viscous phase at $\omega_{kb} = 5$.

4. The partitioning of vorticity into a rigid phase

Considerations given above have shown that there can be three ways of partitioning vorticity in linear viscous bimineralic rocks namely: no partitioning; type 1 partitioning; and type 2 partitioning. The first occurs only at uniform strain rate. When flow occurs in bimineralic rocks at a uniform viscous stress, either type 1 partitioning or type 2 partitioning can occur. In type 1 partitioning the more viscous phase is more rotational, whereas in type 2 partitioning the less viscous phase is more rotational. A geologically interesting situation is the case where one phase in a bimineralic rock is nearly rigid in behaviour, because many metamorphic rocks contain nearly rigid phases such as garnet, cordierite, or feldspar porphyroblasts. The present model can represent partitioning of vorticity in bimineralic rocks in which one phase is rigid by assuming that the viscosity contrast between two phases is infinite. Hence, from Eq. (22) the vorticity partitioning coefficients for type 1 partitioning are obtained as

$$\lim_{b \to \infty} [\alpha_{\mathrm{I}}, \beta_{\mathrm{I}}] = \left[1 - \frac{(1-\phi_1)}{\varpi_{\mathrm{kb}}\phi_1}, \ 1 + \frac{1}{\varpi_{\mathrm{kb}}} \right], \tag{24}$$

and from Eq. (23) those for type 2 partitioning are obtained as

$$\lim_{b \to \infty} [\alpha_{\mathrm{II}}, \beta_{\mathrm{II}}] = \left[1 + \frac{(1-\phi_1)}{\varpi_{\mathrm{kb}}\phi_1}, \ 1 - \frac{1}{\varpi_{\mathrm{kb}}} \right]. \tag{25}$$

It is evident that the vorticity partitioning coefficient for the rigid phase in type 1 partitioning in simple shearing is two and that for type 2 partitioning is zero because the kinematic vorticity number is equal to unity for simple shearing. This means that material of the rigid phase rotates twice as fast as the bulk rock by type 1 partitioning in simple shear but for type 2 partitioning the rigid phase does not rotate at all with respect to the external reference coordinate in simple shear. It should be noted that the external reference coordinate is chosen in such a way that flow becomes simple shear for the bulk rock. In addition, it is shown that one phase may rotate in the opposite direction to the rotation of the bulk rock for certain kinematic vorticity numbers. This condition in type 1 partitioning is given by

$$\varpi_{\mathrm{kb}} < \frac{1-\phi_1}{\phi_1}. \tag{26}$$

Sub-simple shearing ($\varpi_{\mathrm{kb}} < 1$) of the bulk rock results in opposite rotation sense of the less viscous phase in type 1 partitioning if the less viscous phase is less abundant than the rigid phase (i.e. $\phi_{\mathrm{I}} < 0.5$). On the other hand, the condition that the rigid phase rotates in opposite sense to that of the bulk rock (or the less viscous phase) for type 2 partitioning is given by

$$\varpi_{\mathrm{kb}} < 1. \tag{27}$$

Sub-simple shear in the bulk rock usually results in opposite rotation sense of the rigid phase in type 2 partitioning.

5. Discussion

A flow model for ideal linear viscous bimineralic rocks is developed in this paper. The model is an extension of the previous model of Takeda (1998) which formulates the effective viscosity and the partitioning of strain rate between two phases in ideal linear viscous bimineralic rocks. The present model deals with partitioning of vorticity between the two phases.

Two modes of flow behaviour were found to occur, mode 1 and mode 2. Mode 1 is characterized by a linear dependence of bulk rock viscosity on the volume fraction, by lack of partitioning of strain rate (i.e. uniform strain rate) and by lack of partitioning of vorticity (i.e. uniform vorticity). Mode 2 is characterized by a non-linear dependence of bulk rock viscosity on the volume fraction, by uneven partitioning of strain rate (non-uniform strain rate), and by uneven partitioning of vorticity (non-uniform vorticity). Furthermore, two types of partitioning of vorticity were shown to occur in mode 2. One is that the more viscous phase is more rotational than the less viscous phase (type 1 partitioning). The other is that the more viscous phase is less rotational than the less viscous phase (type 2 partitioning). An important conclusion for ideal linear viscous bimineralic rocks is that if partitioning of strain rate between two phases occurs, partitioning of vorticity also occurs.

The effective viscosity of two-phase materials must be greatly dependent on microstructure as well as composition and physical properties of constituent phases. For uniformly mixed and isotropic two-phase materials, however, two bounds can be considered; that is, the Voigt bound and the Reuss bound. The Voigt bound defines the upper limit of effective viscosity of two-phase materials while the Reuss bound defines the lower limit. An actual effective viscosity of two-phase materials may lie somewhere between these two bounds. The Voigt bound is derived under a uniform strain rate condition between the two phases and is expressed by an arithmetical mean weighted by a volume fraction of each phase. The Reuss bound is derived under a uniform stress condition between the two phases and is expressed by an inverse harmonic mean weighted by a volume fraction of each phase. As described above mode 1 behaviour corresponds with the Voigt bound and mode 2 behaviour corresponds with the Ruess bound when densities of the two phases are the same. Hence, we can say that the present model describes the rotational behaviours for the upper bound and the lower bound cases of the bulk effective viscosities.

There are some deformation experiments of bimineralic materials which show that strain is concentrated in a less viscous phase and that the effective viscosity decreases non-linearly with the volume fraction (e.g. Jordan, 1987; Ross et al., 1987). Furthermore, observations of metamorphic, deformed rocks also show that strain concentration occurs in less viscous phases, such as quartz in quartz–feldspar rocks. This implies that partitioning of strain rate occurs among constituent phases in nature. Hence, we may conclude that mode 2 behaviour (lower bound behaviour) is more important in considering a rotational behaviour of flow in natural rocks.

Jeffery (1923) hydrodynamically analyzed the movement of a rigid object in homogeneous and infinitely large viscous media in simple shear flow. Ghosh and Ramberg (1976) applied the theory of Jeffery to the behaviour of a rigid object under shearing with a pure shear component. They showed that the vorticity of a spherical rigid object is identical to the vorticity of the bulk media. The prediction of the theory of Jeffery (1923) for the rotation of a rigid phase may look inconsistent with that of the present model. The present model, however, contains the prediction of the theory of Jeffery as follows. If the vorticity of a rigid phase is equal to that of the bulk media, then, $\omega_b = \omega_1 = \omega_2$. Substituting this relation into Eq. (15), we have

$$D^b_{ij}D^b_{ij} = \phi_1 D^1_{ij}D^1_{ij} + \phi_2 D^2_{ij}D^2_{ij}. \quad (28)$$

Further, additive relation of entropy production rate (or dissipative energy rate) is given by (Takeda, 1998)

$$\mu_b D^b_{ij}D^b_{ij} = \phi_1 \mu_1 D^1_{ij}D^1_{ij} + \phi_2 \mu_2 D^2_{ij}D^2_{ij}. \quad (29)$$

Since a rigid phase is not strained, $D^2_{ij}D^2_{ij} = 0$. Then, from Eqs. (28) and (29) we have

$$\phi_1(\mu_1 - \mu_b)D^1_{ij}D^1_{ij} = 0. \quad (30)$$

It is evident that Eq. (30) is satisfied when $\mu_1 = \mu_b$. The correspondence of the viscosity of the less viscous phase with that of the bulk rock is valid only if a content of the rigid phase is infinitely dilute (i.e. $\phi_2 \to 0$). Accordingly, the present model coincides with the theory of Jeffery (1923) under the special condition that a rigid phase is infinitely dilute.

The situation that one phase is rigid may be geologically realistic because many porphyroblasts in metamorphic rocks appear to behave as rigid phases. However, applicability of the present model to rotational behaviour of flow in natural bimineralic rocks is restricted for two main reasons. One reason is that although Newtonian rheology is assumed in this model, flow in most rocks may deviate significantly from the ideal behaviour. The other reason is that natural rocks may have more complex heterogeneous features than those considered in the present model. The present model is concerned with the properties of constituent phases and the volumetric proportion of the two phases only. It is assumed that each phase is continuous, that is, a continuum in the usual sense. However, phases in natural rocks are actually not continuous on a microscopic scale. For example, a quartz phase in quartz–feldspatic rocks such as a pelitic schist is composed of many grains of quartz of various shapes. Thus, the quartz phase contains complicated grain boundaries in itself. Further, in the present model it is assumed that both the bulk rock and each phase are isotropic. Many minerals are, however, crystallographically anisotropic. In addition, actual rocks often contain various structural anisotropies such as schistosities, crenulation cleavage, layering, and mineral lineations. The effects of such anisotropies and heterogeneities in rocks cannot be assessed by means of the present approach. Furthermore, an evaluation of non-Newtonian behaviour of bimineralic rocks is beyond the scope of this paper. Hence, applicability of the present model to natural rocks is greatly limited. Nevertheless, the present model is a first attempt to provide a theoretical framework that treats the rotational behaviour in polymineralic rocks, and hence, may be considered to be a reference model for more advanced theoretical and experimental studies.

There are currently some controversies in the explanation of the rotational behaviour of porphyroblasts in metamorphic rocks (e.g. Bell, 1985; Bell et al., 1992; Passchier et al., 1992). At present our knowledge of the rheological behaviour of heterogeneous rocks is limited. Furthermore, our theoretical tools, including the present model, are poor for understanding rheological behaviour of actual rocks. The author believes, therefore, that any presumption for rotational behaviour of rigid inclusions in natural rocks is too early to be taken and that more experimental and theoretical investigations must be performed for polymineralic materials.

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